Indirect Temperature Estimation Using Kalman Filter (Two Sensors)

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Interactive Demo: `https://leonathn.github.io/FinalProjectProbability/`

1. Problem Overview

Context: This experimental box was created for Performance Evaluation of Building Environment (Dr. Nguyen Hop Minh). The bulb temperature T_{bulb} is needed to feed CFD (Computational Fluid Dynamics) simulations, but no sensor can measure it directly at 200–300°C without melting. This Kalman Filter method estimates T_{bulb} from indirect air temperature measurements.

Two air-temperature sensors:

Sensor A (near-field)

Close to bulb, high noise: $z_A \approx T_{\text{air-near}}$

Sensor B (far-field)

Farther away, low noise: $z_B \approx T_{\text{air-room}}$

Both are **indirect proxies**. Distances d_A and d_B model heat diffusion.

2. Why Indirect Measurement?

- Bulb surface: **200–300°C** (sensors melt)
- Only air temperature available
- Heat diffusion + convection: noisy, time-varying

⇒ Requires **state estimation**, not direct measurement.

3. Diffusion Model

Heat diffusion model: Temperature attenuates with distance from source.

$$T_{\text{sensor}}(d) = \frac{T_{\text{bulb}}}{1 + d/\ell} + \text{noise}$$

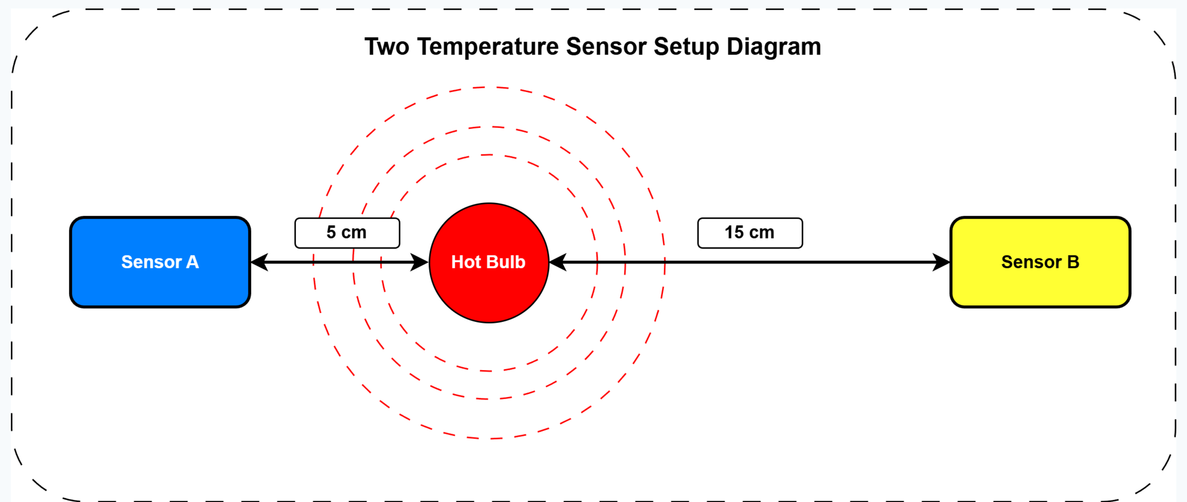
where $\ell = 10$ cm is characteristic length scale.

Airflow, turbulence, convection → measurement noise.

Hidden state: $x = T_{\text{bulb}}$ (cannot measure directly at 200-300°C)

Input data (from CSV):

$$z_A = \text{Middle\_Heat\_Source} \quad (\text{Sensor A, close: } d_A = 5 \text{ cm})$$
$$z_B = \text{Air\_Tube\_Output} \quad (\text{Sensor B, far: } d_B = 15 \text{ cm})$$



Schematic: Sensor A (close, high noise) and Sensor B (far, low noise).

4. Kalman Filter Solution

Linking Diffusion Model to Kalman Filter:

From diffusion model: $z = \frac{x}{1+d/\ell} + v$ relates sensor measurements to hidden bulb temperature via observation model $z = Hx + v$:

- $H_A = 1/(1 + d_A/\ell) = 1/1.5$: Attenuation factor for Sensor A ($d_A = 5$ cm)
- $H_B = 1/(1 + d_B/\ell) = 1/2.5$: Attenuation factor for Sensor B ($d_B = 15$ cm)
- $R_A = 2.0°\text{C}^2$: Sensor A noise variance (close → high turbulence)
- $R_B = 0.5°\text{C}^2$: Sensor B noise variance (far → stable air)
- x: Hidden bulb temperature T_{bulb} (true value unknown)
- $\hat{x}$: Estimated bulb temperature (KF output)
- P: Estimation uncertainty (error covariance)
- $Q = 0.1°\text{C}^2$: Process noise (bulb temperature fluctuations)
- K: Kalman Gain (optimal weight balancing prediction vs measurement)

Probabilistic Framework: Bayesian inference with Gaussian distributions.

Prior: $p(x) = \mathcal{N}(x; \hat{x}, P)$, **Likelihood:** $p(z|x) = \mathcal{N}(z; Hx, R)$

Posterior: By Bayes' rule, $p(x|z) \propto p(z|x) \cdot p(x)$ yields $p(x|z) = \mathcal{N}(x; \mu_{\text{post}}, \sigma^2_{\text{post}})$

The Kalman Filter computes μ_{post} and σ^2_{post} in closed form:

Prediction Step (Prior Propagation)

Purpose: Propagate previous estimate forward in time, accounting for process uncertainty.

$$\hat{x}_{\text{pred}} = \hat{x}_{\text{prev}} \quad \text{(assume bulb temp stays constant)}$$
$$P_{\text{pred}} = P_{\text{prev}} + Q \quad \text{(add process noise: uncertainty grows)}$$

Interpretation: Since we have no control input, the best prediction is the previous estimate. However, uncertainty increases by $Q = 0.1°\text{C}^2$ due to natural temperature fluctuations. This gives prior $p(x) = \mathcal{N}(x; \hat{x}_{\text{pred}}, P_{\text{pred}})$ before incorporating new measurements.

Update Step (Posterior via Bayes)

Purpose: Fuse prediction with sensor measurements to reduce uncertainty.

Sensor A Update: Combine prior $p(x) = \mathcal{N}(x; \hat{x}_{\text{pred}}, P_{\text{pred}})$ with likelihood $p(z_A|x) = \mathcal{N}(z_A; H_A x, R_A)$

$$K_A = \frac{P_{\text{pred}} H_A}{H_A P_{\text{pred}} H_A + R_A} = \frac{P_{\text{pred}}}{P_{\text{pred}} + R_A} \quad \text{(Kalman Gain: optimal weight)}$$
$$\hat{x}_A = \hat{x}_{\text{pred}} + K_A(z_A - H_A \hat{x}_{\text{pred}}) \quad \text{(weighted average of prediction \& measurement)}$$
$$P_A = (1 - K_A H_A) P_{\text{pred}} \quad \text{(uncertainty reduced by measurement)}$$

Key Insight: K_A balances trust in prediction vs measurement:

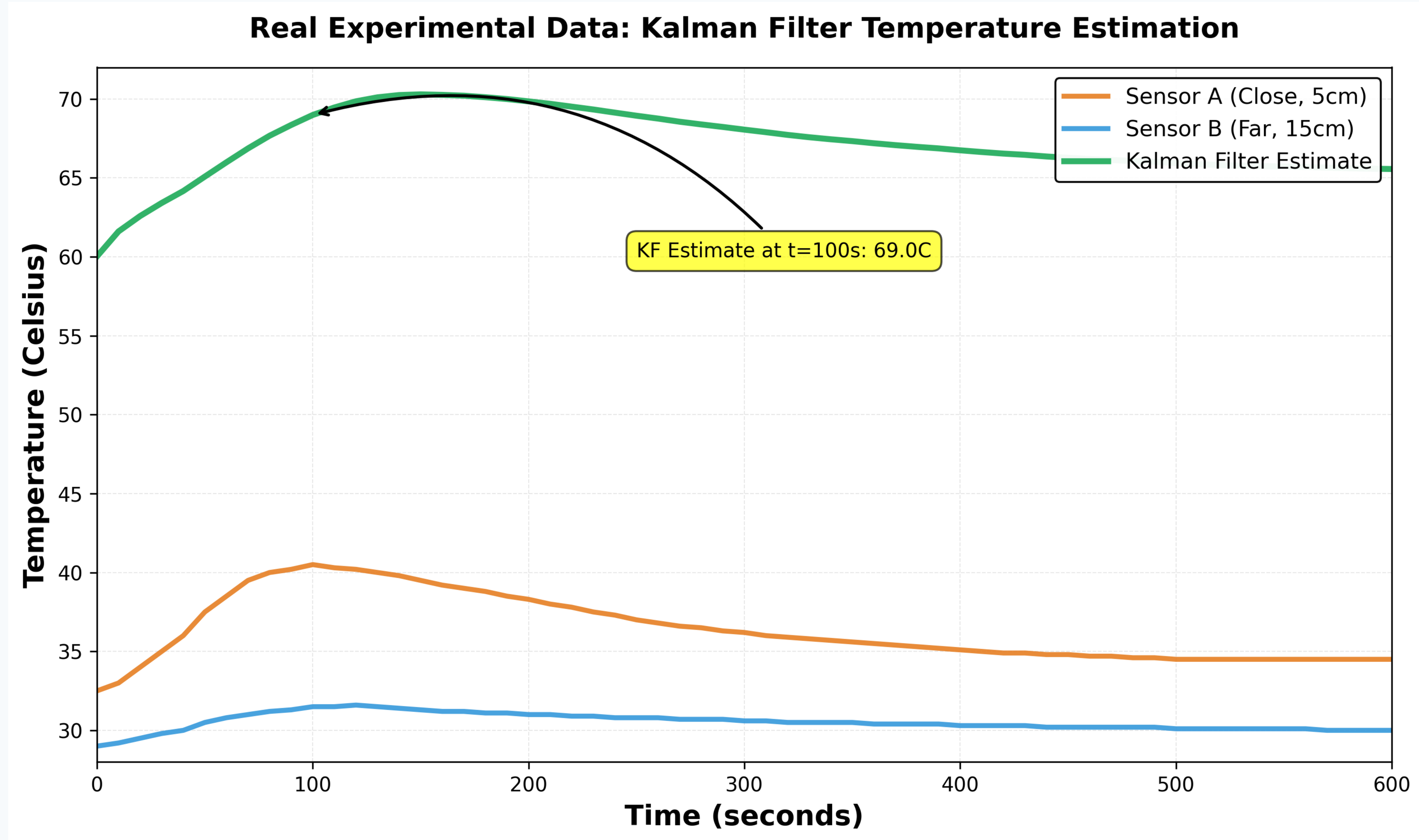
- If $P_{\text{pred}} \gg R_A$ (prediction uncertain, sensor reliable) ⇒ $K_A \approx 1$ (trust sensor)
- If $P_{\text{pred}} \ll R_A$ (prediction confident, sensor noisy) ⇒ $K_A \approx 0$ (trust prediction)

Sensor B Update: Sequentially update with second sensor $p(z_B|x) = \mathcal{N}(z_B; H_B x, R_B)$

$$K_B = \frac{P_A}{P_A + R_B}, \quad \hat{x}_{\text{new}} = \hat{x}_A + K_B(z_B - H_B \hat{x}_A), \quad P_{\text{new}} = (1 - K_B H_B) P_A$$

Result: Final estimate $\hat{x}_{\text{new}}$ optimally fuses both sensors. Since $R_B < R_A$ (Sensor B more reliable), it receives higher weight in fusion.

Real Experimental Data



Setup: Heat source box, two ambient sensors

- $d_A = 5$cm (close), $d_B = 15$cm (far)
- $R_A = 2.0$ (noisier), $R_B = 0.5$ (stable)
- Process noise: $Q = 0.1$

Key Results at $t = 100$s:

- Sensor A: 40.5°C (close, noisy)
- Sensor B: 31.5°C (far, stable)
- **KF bulb estimate: 69.0°C**

Overall Statistics (600s):

- Sensor A: Mean=36.4°C, Std=2.1°C
- Sensor B: Mean=30.5°C, Std=0.6°C
- Bulb estimate: Mean=67.2°C, Std=2.2°C

Data from Dr. Nguyen Hop Minh's Building Environment Performance Evaluation box.

5. Discussion

Method Overview:

- **Challenge:** Estimate hidden bulb temperature from indirect air sensor measurements
- **Approach:** Sequential Bayesian inference via Kalman Filter with two complementary sensors

Experimental Results:

- Bulb estimate: Mean=67.2°C, Std=2.2°C (600s)
- KF fuses Sensor A (close, noisy) with Sensor B (far, stable)
- Kalman Gain adaptively weights sensors by uncertainty

Interactive Demo:

`https://leonathn.github.io/FinalProjectProbability`

6. References

[1] Kalman (1960). *J. Basic Eng.* [2] Welch & Bishop (2006). *UNC.* [3] Simon (2006). *Wiley.*